Write your student exam number in the boxes								

Manly Selective High School

2024 Higher School Certificate Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks:

100

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 41)

- Attempt Questions 11–37
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

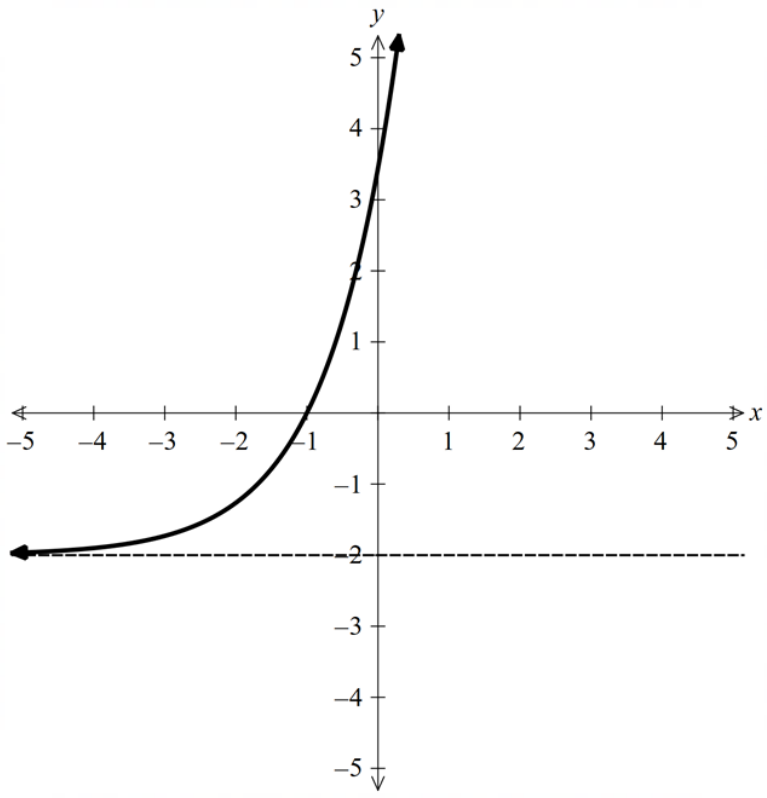
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

1. What is the domain of $f(x) = \sqrt{4x - 8}$?

   A. $x \geq -8$
   B. $x \geq 2$
   C. $x \geq 4$
   D. $x \geq 8$

2. The graph of $y = 2e^{x+b} + c$ is given below.

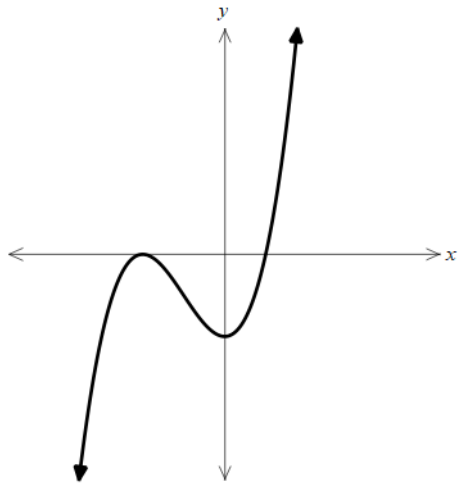


   What is the value of b?

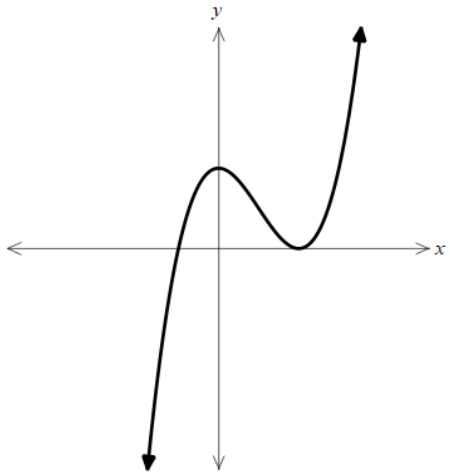
   A. -2
   B. -1
   C. 1
   D. 2

3. Which of the following diagrams shows the graph of $f(x)=(b-x)^2(a+x)$, if $a>0$ and $b>0$?

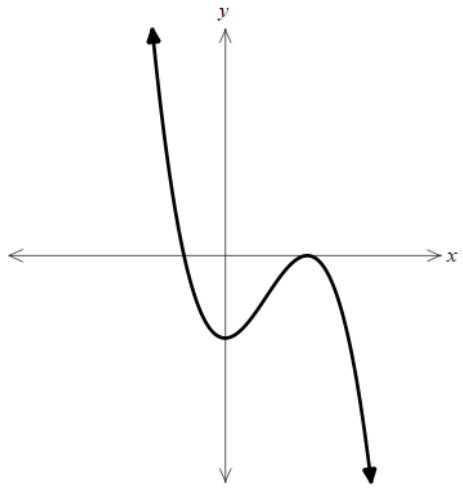
A.



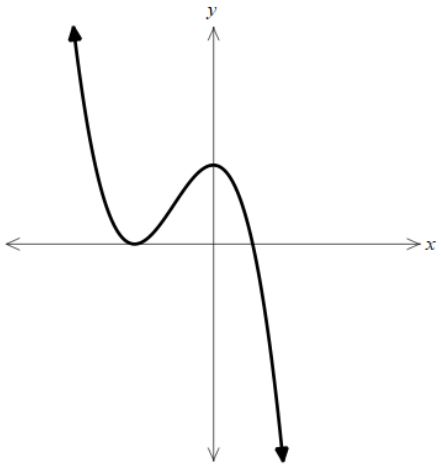
B.



C.



D.



4. A manufacturer of light globes claims that their new globe has a superior lifespan to their old globe.

The box plots below show data for a sample of old and new globes.

Which of the following claims made by the manufacturer is correct?

A. The median lifespan of the new globes is 75% longer than the median lifespan of the old globes.
B. The interquartile range of the new globes is 75% greater than the interquartile range of the old globes.
C. 75% of the old globes last 155 hours or longer.
D. 75% of the new globes last 155 hours or longer.

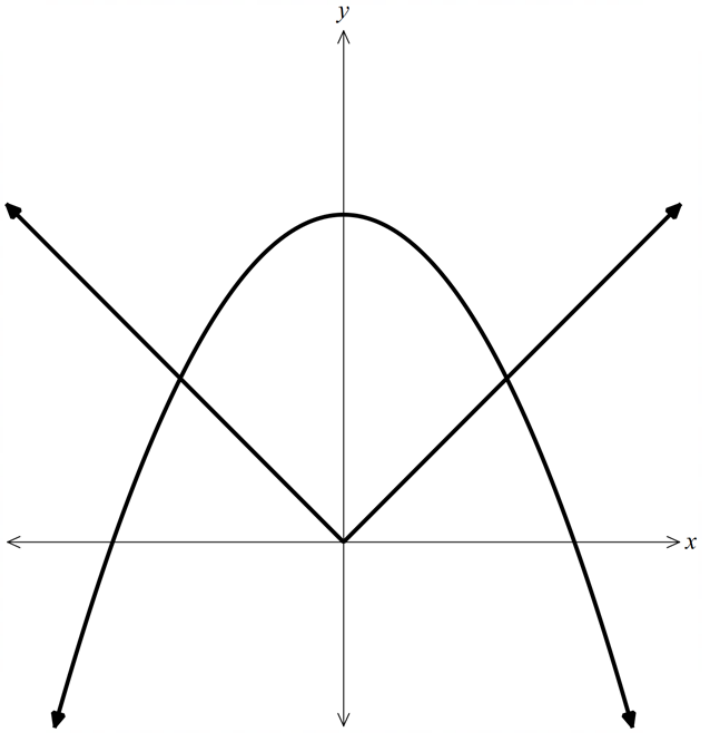
5. Consider the sequence,

$$\log_2 3, \log_2 9, \log_2 27 \ldots$$

What is the sum of the first 10 terms of this sequence?

A. $3\log_2 3$
B. $10\log_2 3$
C. $30\log_2 3$
D. $55\log_2 3$

6. The diagram below shows the curves $y = |x|$ and $y = 2 - x^2$.



What is the area between the two curves?

A. $\frac{7}{6}$ units^2

B. $\frac{14}{3}$ units^2

C. $\frac{7}{3}$ units^2

D. $\frac{13}{3}$ units^2

7. For two functions $g(x)$ and $h(x)$, it is known that for all $a > 0$, $g(a) > h(a)$.

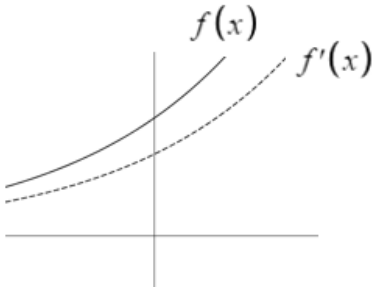
It is also known that $h'(x) < 0$ and $g'(x) > 0$ for $x > 0$.

How many solutions are there to $g(x) - h(x) = 0$ in the domain $a > 0$?

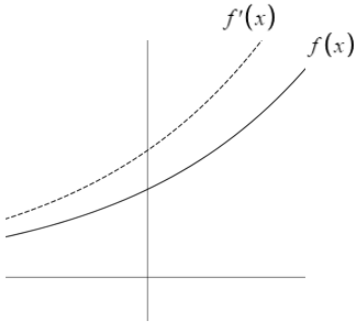
A. 0
B. 1
C. 2
D. 3

8. Which of the following shows the graph of $f(x) = 2^x$ and its derivative on the same plane?

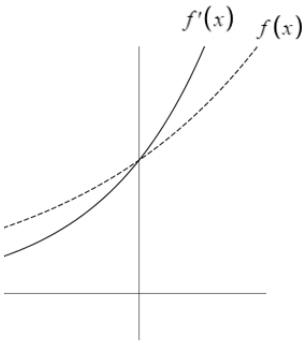
A.



B.



C.



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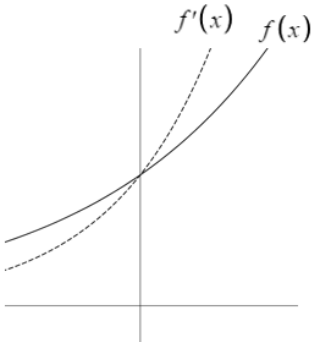

9. If $f(x) = x(x-1)$ how many different solutions are there to the equation $f(x^2) = 0$?

A. 1

B. 2

C. 3

D. 4

10. Let $f(x) = k\sin(mx + l)$, $m > 0$ where $f(x)$ is continuous and increasing for $a \leq x \leq b$, $f(a) = -k$ and $f(b) = k$.

Let $g(x) = k\cos(mx + l)$.

Which of the following statements is true for $g(x)$ in the domain $a \leq x \leq b$?

A. $g(x)$ is always increasing
B. $g(x)$ is always decreasing
C. There exists a value c such that $g(c) = k$
D. There exists a value c such that $g(c) = -k$

End of Section I

2024 Higher School Certificate Trial Examination
NBSC Manly Campus

Write your student exam number in the boxes								

Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks
Attempt questions 11 – 37.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 19 (22 marks)
Booklet 2 – Attempt Questions 20 – 24 (23 marks)
Booklet 3 – Attempt Questions 25 – 30 (22 marks)
Booklet 4 – Attempt Questions 31 – 37 (23 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks)

It is given that y is inversely proportional to x. When $x = 8$, $y = 2$.

(a) Find the equation relating x and y. **1**

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(b) Graph the relationship showing two different points on the graph. **2**

Question 12 (2 marks)

The domains of two continuous functions, $f(x)$ and $g(x)$, are $[-3, 2)$ and $(0, 6]$ respectively.

What is the domain of the sum function $f(x) + g(x)$? **2**

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Question 13 (4 marks)

Let $f(x) = (x-3)^2 - 1$.

$f(x)$ is transformed to $g(x)$ such that $g(x) = \frac{1}{2} f(2-x)$.

(a) Complete a possible sequence of transformations that maps $f(x)$ to $g(x)$. **2**

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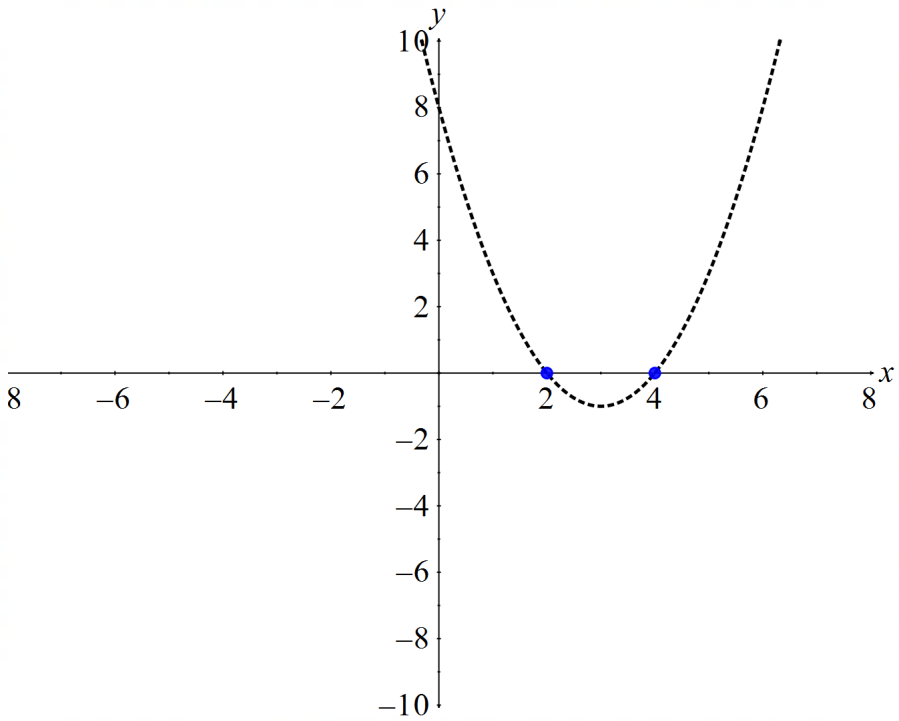
(b) On the same set of axes below, sketch the graph of $y = g(x)$ below.



Question 14 (3 marks)

The table below shows the future value of an annuity with contributions of $1 at the end of each period, at different interest rates over different numbers of time periods.

Time Period	**Interest rate**				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(a) What amount of money needs to be contributed at the end of each year to provide $315 932 after 7 years at 4% per annum compound interest? **1**

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(b) An amount of $5000 is invested every six months, at 4% per annum, compounded every six months for 3 years. What is the amount of interest earned? **2**

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Question 15 (2 marks)

Solve $\log_2(x + 3) - \log_2 x = 1$ **2**

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Question 16 (2 marks)

An investment of \$4000 earns interest at a rate of r% p.a. over a period of one year. How much more interest would the investment earn, if interest is compounded half-yearly rather than yearly? Find the answer in terms of r. **2**

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Question 17 (2 marks)

Solve $2\ sin^2 x - 1 = 0$ for $-180° \leq x \leq 180°$ **2**

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Question 18 (2 marks)

Find the equation of the tangent to the curve $y = (2 - x)^4$ at the point $(3\,,\,1)$. **2**

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Question 19 (2 marks)

Given $\tan\alpha = -0.2$ and $\sin\alpha > 0$, find the exact value of $\cos\alpha$. **2**

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End of Booklet 1

2024 Higher School Certificate Trial Examination
NBSC Manly Campus

Write your student exam number in the boxes								

Mathematics Advanced

Section II Answer Booklet 2

Section II

90 marks
Attempt questions 11 – 37.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 19 (22 marks)
Booklet 2 – Attempt Questions 20 – 24 (23 marks)
Booklet 3 – Attempt Questions 25 – 30 (22 marks)
Booklet 4 – Attempt Questions 31 – 37 (23 marks)

Instructions

- Write your Student Number at the top of this page.
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Question 20 (4 marks)

Find the global maximum of the function $y = x^2(x+2)^3$ for $-3 \leq x \leq 1$.

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Question 21 (4 marks)

(a) Differentiate $\sqrt{3+x^3}$. **2**

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(b) Hence, find $\int \frac{x^2}{\sqrt{3+x^3}}dx$. **2**

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Question 22 (3 marks)

Evaluate $\int_1^3 \frac{24x^2-14}{4x^3-7x+5}$ leaving your answer in simplest exact form. **3**

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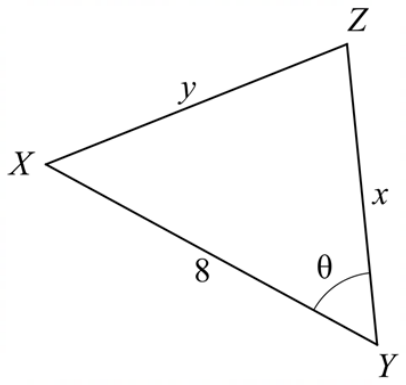
Question 23 (8 marks)

Triangle XYZ has a perimeter of 20 cm.



(a) Show that $\cos\theta = \frac{3x-10}{2x}$. **2**

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(b) If the area of the triangle is A, show that $A^2 = -20(x^2 - 12x + 20)$. **3**

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Question continued on the next page.

(c) Find the values of x and y when the area is a maximum. **3**

Question 24 (4 marks)

(a) Show that $\frac{\text{cosec } A}{\text{cosec } A - \sin A} \equiv sec^2 A$. **2**

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(b) Hence, or otherwise, find $\int \tan^2 x \left(\frac{\text{cosec } x}{\text{cosec } x - \sin x} \right) dx$.

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End of Booklet 2

2024 Higher School Certificate Trial Examination
NBSC Manly Campus

Write your student exam number in the boxes								

Mathematics Advanced

Section II Answer Booklet 3

Section II

90 marks
Attempt questions 11 – 37.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 19 (22 marks)
Booklet 2 – Attempt Questions 20 – 24 (23 marks)
Booklet 3 – Attempt Questions 25 – 30 (22 marks)
Booklet 4 – Attempt Questions 31 – 37 (23 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Question 25 (3 marks)

A polynomial function $f(x)$ has the following properties.

	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<3$	$x=3$	$x>3$
$f'(x)$	<0	0	>0	>0	>0	0	>0
$f''(x)$	>0	>0	>0	0	<0	0	>0

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Sketch a possible graph of $y=f(x)$, using the properties in the table above.

Question 26 (2 marks)

Let $\log_b x = \frac{1}{3}$ and $\log_b y = \frac{1}{4}$. Find the value of $\log_b\left(b^3\frac{\sqrt{x}}{y}\right)$. **3**

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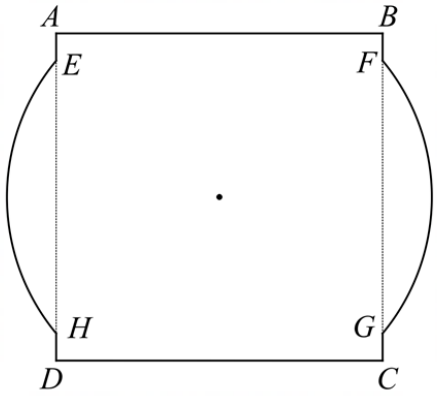
Question 27 (3 marks)

The diagram shows a shape that consists of a square with two identical arcs on opposite sides of the square.



The square has a side length of 12m and the arcs have their centre at the centre of the square.

The points E, F, G and H are the endpoints of the arcs.

AE = FB = CG = HD = 1 m.

Calculate the area of the shape, to the nearest m^2. **3**

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Question 28 (6 marks)

Preeti deposits $300 into a savings account on 1 January 2024. The account accrues interest at the rate of 3.6% p.a. compounding monthly. Preeti plans to deposit $300 at the start of every month thereafter until the end of 2029.
Let A_n be the amount that Preeti has in her account at the end of the nth month (where the 1st month is January 2024).

(a) By first obtaining expressions for A_1 , A_2 , and A_3 , show that $A_n = 100300(1.003^n - 1)$ **3**

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Question continued on the next page

(b) Preeti quits her job and goes to university for two years starting in 2028. As a result, she is unable to make any monthly deposits in the years 2028-2029. However, while she is at university, her bank offers her a special interest rate of 6% p.a. compounding monthly.

Calculate the value of Preeti's account at the end of 2029, and find how much more money she would have had if she hadn't gone to university. **3**

Question 29 (3 marks)

A random variable is defined by the probability distribution:

$$P(X = x) = \frac{x^2 + x}{20} \text{ for } x = 1, 2, 3.$$

Find the value of $Var(X)$.

3

Question 30 (4 marks)

Two events A and B are independent, with $P(A) = p$, $P(B) = 2p - \frac{5}{6}$ and $P\left(\overline{A} \cap \overline{B}\right) = p^2 - \frac{5}{18}$. **4**

Find the value of p.

End of Booklet 3

2024 Higher School Certificate Trial Examination
NBSC Manly Campus

Write your student exam number in the boxes								

Mathematics Advanced

Section II Answer Booklet 4

Section II

90 marks
Attempt questions 11 – 37.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 19 (22 marks)
Booklet 2 – Attempt Questions 20 – 24 (23 marks)
Booklet 3 – Attempt Questions 25 – 30 (22 marks)
Booklet 4 – Attempt Questions 31 – 37 (23 marks)

Instructions

- Write your Student Number at the top of this page.
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- Your responses should include relevant mathematical reasoning and/or calculations.
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Question 31 (2 marks)

In a particular board game, a six-sided die is rolled to determine how a player moves their game piece around the board. **2**

If the number rolled is odd, the player moves their game piece forward the number of spaces shown on the die.

If the number rolled is even, the player does not move.

Calculate the expected number of spaces moved each turn.

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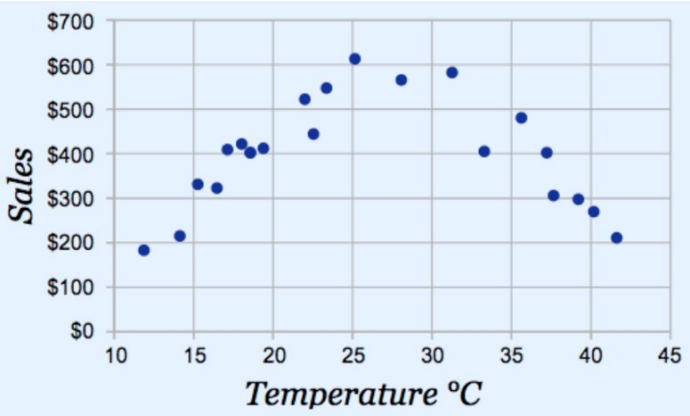
Question 32 (3 marks)

Mei owns a shop that sells smoothies (a type of cold drink). The scatterplot below shows the shop's smoothie sales on 21 different days in 2024. The maximum temperature on each day is also shown.



Mei entered the data into a spreadsheet and calculated the following:

Pearson's correlation coefficient: $r = 0$
Least-squares regression line: $y = 390$

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Explain the meaning of the correlation coefficient and the least squares regression line with reference to *Sales* and *Temperature*.

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Question 33 (3 marks)

A company manufactures cans of soft drink. The volume inside each can is normally distributed with a mean of 375mL and a standard deviation of 3mL. It is also known that approximately 99% of the cans have a volume of less than 382mL.

During the quality control process, cans are rejected if they are underfilled or overfilled by a certain amount. 1% of the cans are rejected for being underfilled and 2.5% of the cans are rejected for being overfilled. **3**

Between which two volumes do the cans that are not rejected lie?

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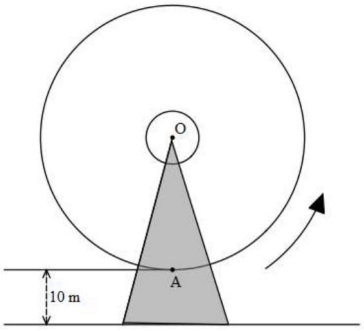
Question 34 (5 marks)

An observation wheel has its centre at point O. The wheel moves anticlockwise at a constant speed and completes one full rotation every 30 minutes.

When the wheel is at its lowest point (A), it is 10 metres above ground.

The radius of the wheel is 50 metres.



The height above ground of a carriage pod initially at point A can be modelled by the function $h(t) = k\cos(bt) + c$ after time t minutes.

(a) Find the values of k, b and c. **2**

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Question continued on the next page

(b) Passengers will have a view of the ocean when the carriage pod reaches a height of 85 metres or greater above ground.

Thilara boards the carriage pod at 12.30pm, hoping to see the ocean as much as possible before exiting the carriage pod at 1.30pm.

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At what times of the day will Thilara be able to see the ocean?

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Question 35 (3 marks)

At a bus stop on a particular day, a bus arrives every 10 minutes. The time spent waiting for the next bus (X minutes) is a continuous random variable with the probability density function:

$$f(x) = \begin{cases} k & x \in [0,10] \\ 0 & x \in (-\infty,0) \cup (10,\infty) \end{cases}$$

Kenta goes to the bus stop twice, once in the morning and again in the afternoon.

What is the probability that at least one of his buses will require a wait time of more than 7 minutes?

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Question 36 (3 marks)

The diagram shows the graph of the curve $y = x^2 - 1$ for $0 \leq x \leq k$ where $k > 1$.

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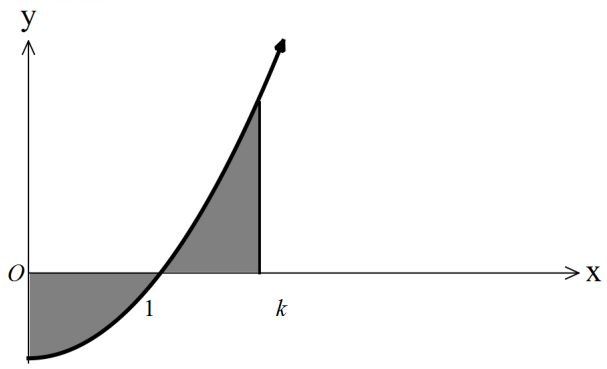

The areas of the two shaded regions are equal. Find, in simplest exact form, the value of k.

Question 37 (4 marks)

An arithmetic and geometric sequence both have first term 1, and their second terms are equal.

The 14^{th} term of the arithmetic sequence is three times the third term of the geometric sequence.

Given that both sequences contain only integers, find the common ratio and common difference for each sequence. **4**

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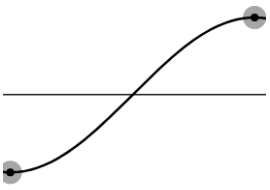
End of examination

Q	2024 Advanced ManlyCampus Trial HSC Solutions	Marking Guidelines
1.	$4x - 8 \geq 0$ $x \geq 2$	B
2.	The value of c is -2 (asymptote). $y = 2e^{x+b} - 2$ The curve passes through (-1, 0) $0 = 2e^{b-1} - 2$ $2e^{b-1} = 2$ $e^{b-1} = 1$ $b - 1 = 0$ $b = 1$	C
3.	Curve must cross the x-axis at $x = -a$ and bounce off the x-axis at $x = b$	B
4.	Claim A: median of new globes (185) is 75% greater than median of old globes (150) (incorrect) Claim B: IQR of new globes (45) is 75% greater than IQR of old globes (25) (incorrect) Claim C: 75% of old globes last 155 hours or longer. But Q1 of old globes is 130. (incorrect) Claim D: 75% of old globes last 155 hours of longer. Q1 = 155, so this is correct.	D
5.	$\log_2 3 , \log_2 9 , \log_2 27 , ...$ $= \log_2 3 , 2\log_2 3 , 3\log_2 3 , ...$ Arithmetic sequence with $a = \log_2 3$, $d = \log_2 3$ $S_{10} = \frac{10}{2}\left[2(\log_2 3) + 9(\log_2 3)\right]$ $= 5(11\log_2 3)$ $= 55 \log_2 3$	D
6.		C

7.	From a sketch or by understanding the Information:	A
8.	From Reference sheet	A
9.	$f(x^2) = x^2(x^2 - 1) = 0$ $x^2 = 0, 1$ $x = 0, \pm 1$ So there are three different solutions to the equation.	C
10.	**Explanation 1:** As $f(a) = -k$ and $f(b) = k$ and $f(x)$ is continuous, then there exists a value such that $f(c) = 0$, where $a < c < b$. Differentiating $g'(x) = -km\sin(mx + \ell)$, $\therefore g'(x) = -mf(x)$. This implies $g(x)$ has a stationary point between $a < x < b$ as $f(x) < 0$ and then $f(x) > 0$ this implies $g'(x) < 0$ and then $g'(x) > 0$, which means there exists a maximum stationary point. i.e. max occurs at the amplitude of $g(x)$ $g(c) = k$ C **Explanation 2:** $f(x)$ looks like this in the interval $[a, b]$: 	C

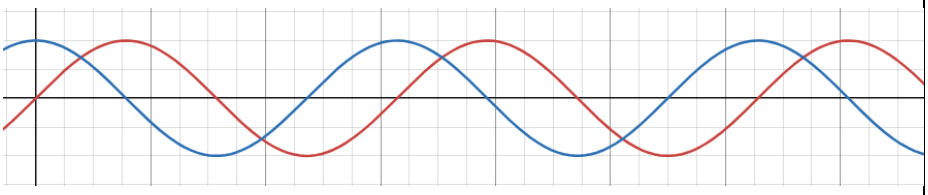
$g'(x) = -km\sin(mx + l) = -km \times f(x)$
Since $f(x)$ changes sign in the interval $[a, b]$, $g'(x)$ must also change sign, therefore $g(x)$ is not always increasing or decreasing [so A and B are incorrect].

Consider solutions to the equation $g(x) = \pm k$
$k\cos(mx + l) = \pm k$
$\cos(mx + l) = \pm 1$

Consider the sine and cosine graphs:

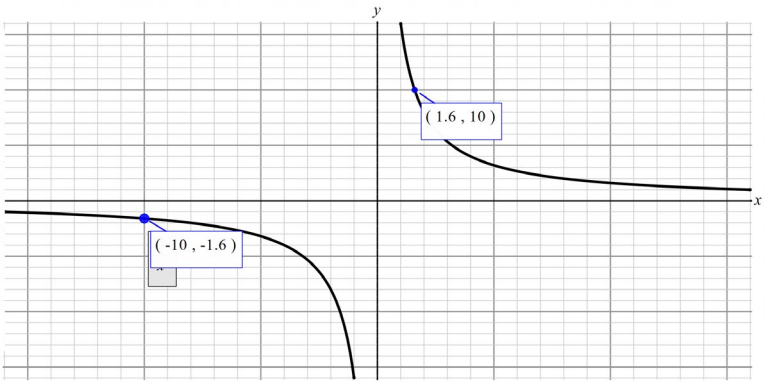
$\cos(mx + l) = 1$ when $mx + l = 0, 2\pi, 4\pi, 6\pi$ (the even multiples of π)
i.e. whenever $y = \sin(mx + l)$ is increasing through the x-axis.

$\cos(mx + l) = -1$ when $mx + l = \pi, 3\pi, 5\pi$ (the odd multiples of π)
i.e. whenever $y = \sin(mx + l)$ is decreasing through the x-axis.



Since $f(x)$ has only one x-intercept and is increasing there, $g(x) = k$ has a solution [so C is correct] and $g(x) = -k$ doesn't [so D is incorrect].

Booklet 1

11	a) $xy = k$ $8 \times 2 = 16$ $\therefore y = \frac{16}{x}$  Only correct answers from part a received marks.	1 mark for correct equation. Many solutions had taken the relationship as direct proportion. As this simplifies the question, no mark was given here or in part b below. b) 1 for shape 1 for the two different points. Some graphs had both points on one branch of the hyperbola which did not cost them marks. Best to have one point on each branch. Students are advised to be careful with their graphs near the asymptotes.
12	Since the sum requires intersection, the domain is $(0,2)$	1 mark for each of the boundaries. Some erroneous solutions gave the union of the two domains.
13	a) Reflect about the y-axis translate 2 units to the right vertical dilate by factor of ½ OR **Question 1 (**Translate 4 units to the left **Question 2 (**Vertically dilate by ½ b) Only correct answers from the given equation in the question received marks.	1 mark for partial transformation steps given. 2 marks for either of these samples. Some solutions had the incorrect direction, or dilation factor, which cost one mark. b) required correct intercepts and minimum turning point that should be higher than the graph's minimum. Since the question was asking to sketch the graph with the equation given, no carry errors were given.
14	a) $\frac{315932}{7.8983} = \$40000$	1 mark for correct answer, or equivalent expression using the number 7.8983 from the table.

	b) $5000 \times 6.3081 - 5000 \times 6 = \$1514.50$	Solutions that used series were accepted as use of table was not emphasised. Part b) gained one mark for the correct number from the table given every six months. Second mark for subtracting the invested amount for finding the amount of interest. Using series, if correctly done, also gained marks as appropriate.
15	$log_2^{(x+3)} - log_2^x = 1$ $log_2^{\frac{x+3}{x}} = 1$ $2^1 = \frac{x+3}{x}$ $2x = x + 3$ $x = 3$	Mostly done well, with the linear equation solved correctly. Some solutions incorporated log laws incorrectly that did not gain marks. 1 mark for correct log law. 2 marks for correct working out to get to $x = 3$
16	Investment with yearly interest = $4000(1 + \frac{r}{100})$ Investment with half-yearly interest = $4000\left(1 + \frac{r}{200}\right)^2$ $= 4000(1 + \frac{r}{100} + \frac{r^2}{40000})$ Extra earning = $4000\left(1 + \frac{r}{100} + \frac{r^2}{40000}\right) - 4000\left(1 + \frac{r}{100}\right) =$ $4000\left(\frac{r^2}{40000}\right) = \frac{r^2}{10}$	Many students recognised that the interest rate is halved for half yearly case. The better solutions considered r as a percentage, $\frac{r}{100}$ in the calculations. Some solutions considered *n* number of terms, which did not get to any results.
17	$2\sin^2 x = 1$ $\sin^2 x = \frac{1}{2}$ $sinx = \pm\frac{1}{\sqrt{2}}$ $\therefore x = \pm 45, \pm 135$	Mostly done well. Some solutions only gave the positive value of $sinx$ which gained only one mark. Showing the +/- for the solution gained one mark and stating the four solutions gained the other mark.
18	$y' = 4 \times (2 - x)^3 \times (-1) = -4(2 - x)^3$ When $x = 3$ $m = -4(2 - 3)^3 = 4$ $\therefore y - 1 = 4(x - 3)$ $y = 4x - 11$	Mostly done well. Some errors occurred from taking $x = 1$ & $y = 3$

		Some solutions mistook the tangent and normal's gradient. 1 mark was awarded for the correct working out for the gradient and 1 mark was awarded for the correct working out to the equation of the tangent.
19	If $tan\alpha < 0$ & $sin\alpha > 0$ then the angle is in the second quadrant, hence negative value for $cos\alpha$ From the given ratio Hence, $cosa = -\frac{5}{\sqrt{26}}$ Only correct answers from received marks for the sign and value.	Most complete solutions used the ratio of the sides and Pythagoras to find the hypothenuse and hence the value of $cosa$. Some students forgot to factor the sign of $cosa$. 1 mark for correct working out for finding the value and 1 mark for the sign of the answer.

General notes:

Write numbers and letters clear, such as 2, z, 7, 3, 5, s, r…

When using extra space, indicate in your paper.

If a table or graph is given, use it in your solution.

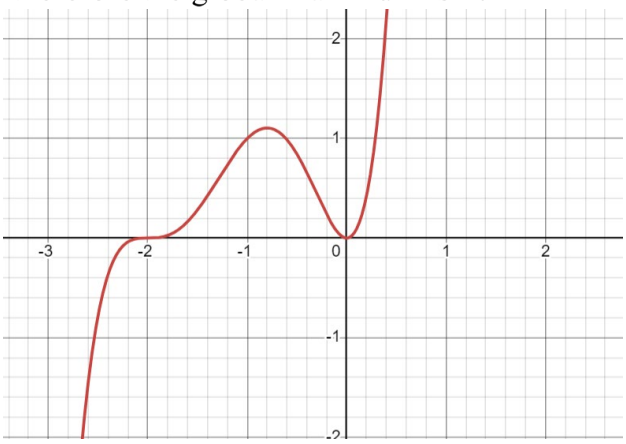
Sketch with a pencil, then go over with a smooth pen stroke.

Asymptotes do not have to be as long as the page. The graph does not have to be parallel to the asymptote.

Exact value requires surds to be whole numbers.

If giving equation of a line in general form, know what should it look like.

Booklet 2

20	$y' = 2x(x+2)^3 + 3(x+2)^2 x^2$ $= x(x+2)^2(2x+4+3x)$ $= x(x+2)^2(5x+4)$ $= 0$ when $x = 0, x = -2, x = -0.8$ From the graph $x = -2$ is a horizontal point of inflexion and $x = 0$ is a min turning point. Therefore we must test $x = -0.8$ and the endpoints. when $x = -0.8$ $y = 1.10592$ when $x = -3$ $y = -9$ when $x = 1$ $y = 27$ therefore the global maximum is 27  	4 marks correct working and solution. 3 marks correct working however did not state the global maximum or did not use the correct reasoning for turning points or equivalent. 2 marks correct working however only uses the stationary points to state the global maximum or equivalent. Did not test the endpoints and did not state the global maximum. 1 mark correct derivative or tests endpoints, but make no further progress.
21a	$y = (3+x^3)^{\frac{1}{2}}$ $y' = \frac{1}{2}(3+x^3)^{-\frac{1}{2}} \times 3x^2$ $y' = \frac{3}{2\left(\sqrt{3}+x^3\right)}$	2 marks for correct solution. 1 mark for correct use of chain rule however did not simplify.
21b	$\int \frac{x^2}{\sqrt{3+x^3}}\,dx$ $= \frac{2}{3}\int \frac{3x^2}{2\sqrt{3+x^3}}\,dx$ $= \frac{2\sqrt{3+x^3}}{3} + C$	2 marks correct working and solution. 1 mark for multiplying integral by 3/2 and 2/3 or equivalent.

22	$= 2\int_1^3 \frac{12x^2 - 7}{4x^3 - 7x + 5}\, dx$ $= 2\ln\ \lvert 4x^3 - 7x + 5 \rvert \Big\rvert_1^3$ $= 2(\ln\lvert 92 \rvert - \ln\lvert 2 \rvert)$ $= 2ln46$	3 marks for correct working and solution. 2 marks for correct integral however incorrect evaluation. 1 mark for factoring out a 2 from the numerator.
23a	$\cos\theta = \frac{8^2 + x^2 - y^2}{2 \times 8 \times x}$ $\cos\theta = \frac{64 + x^2 - (12 - x)^2}{16x}$ $\cos\theta = \frac{64 + x^2 - 144 + 24x - x^2}{16x}$ $\cos\theta = \frac{24x - 80}{16x}$ $\cos\theta = \frac{3x - 10}{2x}$	2 marks for correct working and solution 1 mark for correct use of cosine rule and substitution of y = 12 – x, however no further progress is made.

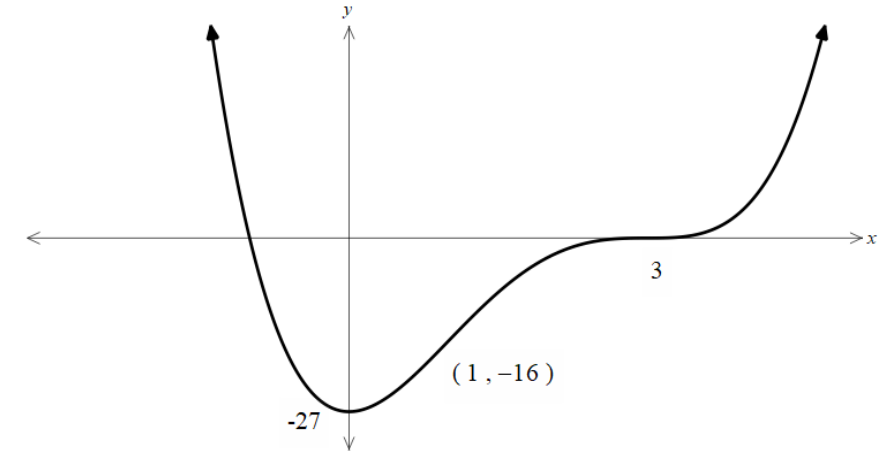
23b	$A = \frac{1}{2} \times 8 \times x \times \sin\theta$ $A^2 = \frac{1}{4} \times 64 \times x^2 \times \sin^2\theta$ $A^2 = 16x^2(1 - \cos^2\theta)$ $A^2 = 16x^2 - 16\cos^2\theta$ $A^2 = 16x^2 - 16 \times \frac{(3x-10)^2}{4x^2}$ $A^2 = 16x^2 - 4x^2(9x^2 - 60x + 100)$ $A^2 = 16x^2 - 36x^2 + 240x + 400$ $A^2 = -20x^2 + 240x + 400$ $A^2 = -20(x^2 - 12x + 20)$	3 marks correct working and solution 2 marks for correct use of area formula and substitution of cos squared theta but does not progress to the required solution. 1 mark for A^2 in terms of sin theta.
23c	Let $A^2 = f(x)$ $f'(x) = -40x + 240$ $f'(x) = 0$ when $x = 6$ $f''(x) = -40 \therefore$ a maximum occurs at $x = 6$ when $x = 6$ $y = 6$ NOTE: students who differentiated A were still awarded full marks if everything was correct. However, this is not necessary. Students who also stated that this is a concave down parabola and stated the axis of symmetry and the max value were also awarded full marks if everything was correct.	3 marks for correct working and solution. 2 marks for finding the correct values of x and y, however did not provide a reason as to why a maximum occurs. 1 mark for differentiating A or A^2 or stating concave down parabola, but makes no further progress.
24a	$\text{LHS} = \frac{\text{cosecA}}{\text{cosecA} - \text{sinA}}$ $= \frac{\frac{1}{sinA}}{\frac{1}{sinA} - sinA}$ $= \frac{1}{1 - sin^2A}$ $= \frac{1}{cos^2A}$ $= sec^2A$	2 marks for correct working and solution. 1 mark for writing LHS in terms of sinA
24b	$\int tan^2x\, sec^2x\, dx$ $= \frac{1}{3}tan^3x + C$	2 marks for correct working and solution. 1 mark for substituting sec squared x.

	Integral Calculus $\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$ where $n \neq -1$ Using from reference sheet	

General Notes

- Common mistakes were:
- * Q20 on not testing the endpoints
- * Q23 b, not understanding the algebra being used and the substitution of $\sin^2$ and $\cos^2$
- * Q23 c students not providing a reason as to why the area is a maximum
- Q24 b students not using the reference sheet.

Booklet 3

Q 25	From table 	1 mk each but minus 1 for each error Must have correct shape Min stat pt on the y axis Pt of Inflection at x=1 Graph needs to show change of concavity Horizontal POI at x=3. Graph needs to be horizontal at x=3 and show change of concavity
26		3 marks correct solution 2 marks for 2 correct applications of log laws 1 mark for one part correct for correct reason

27		3 marks correct solution 2 marks for correct approach with area of sector minus triangle and angle at centre calculated correctly 1 mark for correct angle at centre Many incorrect assumptions and approaches that made the qn easier but incorrect. Only 1 mark available for this mistake
28	a) Deposits $300 at start of each month  b) After 4 years she has made 48 payments  If she had stayed at her job for 6 years n=72	3 marks for showing correct An value with A1, A2, A3 expressions 2 marks correct A1, A2 and A3 expressions but failed to show An 1 mark for partially correct for expressions 3 marks correct solution 2 marks for 2 parts correct but errors 1 mark for 1 part correct Many mistakes in the time

		periods causing reduced marks
29		3 marks correct solution 2 marks for correct mean and partially correct Variance 1 mark for mean or carry error to Var(X) if mean incorrect

30	 	4 marks correct solution 3 marks correct solution but failed to discount invalid p value 2 marks for correct equation but incorrect solution 1 mark for correct application of Independent events Many students stated incorrect probability facts and hence were NOT able to access more than 2 marks

	$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$ $p^2 - \frac{5}{18} = (1-p)\left(1 - 2p + \frac{5}{6}\right)$ $p^2 - \frac{5}{18} = (1-p)\left(\frac{11}{6} - 2p\right)$ $p^2 - \frac{5}{18} = \frac{11}{6} - \frac{23p}{6} + 2p^2$ $p^2 - \frac{23p}{6} + \frac{19}{9} = 0$ $18p^2 - 69p + 38 = 0$ $p = 69 \pm \frac{\sqrt{(-69)^2 - 4 \times 18 \times 38}}{2 \times 18}$ $= \frac{69 \pm 45}{36}$ $= \frac{114}{36}$ or $\frac{24}{36}$ $= \frac{19}{6}$ or $\frac{2}{3}$ But $0 \leq p \leq 1$ $\therefore \quad p = \frac{2}{3}$ only	

General Notes:

Graph in Qn 25 was very poorly drawn. Should label the key points

Log laws poorly known in Q26

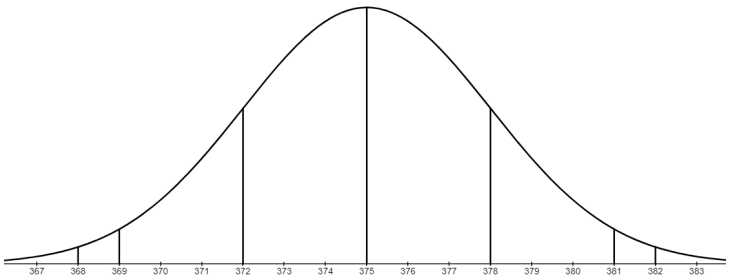
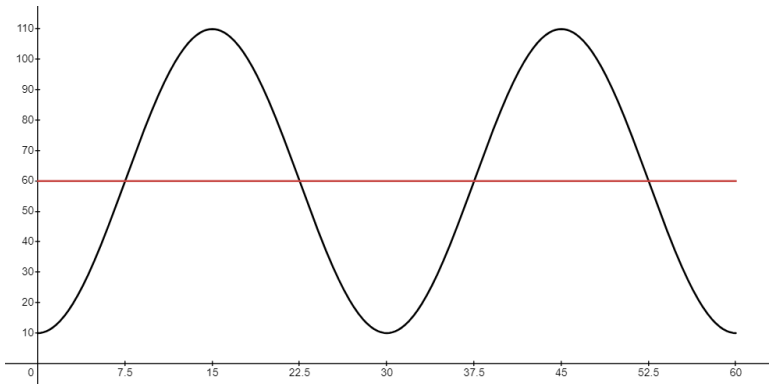
Q27 Very badly set out with poor working. Approach of doing a circle minus square was not a valid approach.

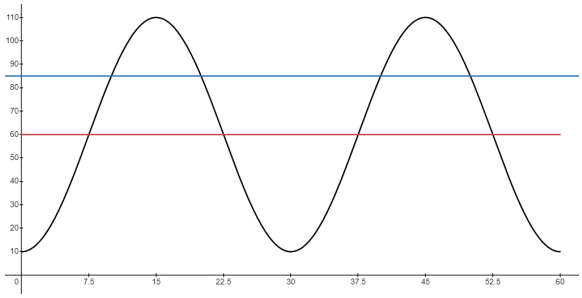
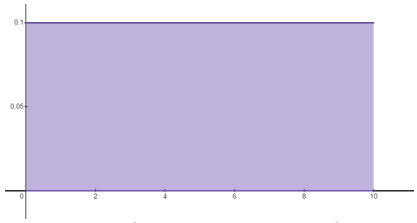
Q8 a) Students failed to read question correctly or failed to apply CI to all investments then failed to SHOW the An value given

c) Some students used incorrect expression for An using their own rather than given formula. This resulted in wrong answers

Booklet 4

31	Let X be the number of spaces moved forward: | x | 0 | 1 | 3 | 5 | | --- | --- | --- | --- | --- | | $P(X=x)$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $E(X) = (0)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right)$ $= \frac{3}{2}$	2 marks 1 mark for a valid probability distribution table
32	1) r = 0 indicates no (linear) correlation between the variables. [Saying.»no.correlation‹ .receives.7.mark] 2) This doesn't mean that there is no correlation whatsoever – we can see a non-linear (possibly parabolic) trend whereby Temperature increases up to a certain point (25 degrees) then decreases. This is possibly because at cool temperatures there is less of a demand for smoothies, and at very high temperatures not many people are going out to the shop. [Describing.the.»non_linear‹ .trend receives.7.mark¡.e¡g¡.Sales. increases.then.decreases.–.Sales.vs¡.Temperature.looks. parabolic] 3) If the trend must be modelled by a straight line, then y = 390 is the best possible line. It is a horizontal line through the middle of the points, with about half of the points below the line and half above. $390 represents the 'average' Sales for a given day. Clearly a straight line is unable to capture the 'increasing then decreasing' trend [Saying.y±966.is.the.average.value.–.centre.of.the.points. receives.7.mark]	3 marks: 1 mark for each point

33	 $P(X < 382) = 0.99$ (given) $\Rightarrow$ $P(X > 368) = 0.99$ (symmetry) $\Rightarrow$ $P(X \leq 368) = 0.01$, so 368 is the minimum acceptable $P(369 \leq X \leq 381) = 0.95$ (empirical rule) $\Rightarrow$ $P(X < 369$ or $X > 381) = 0.05$ $\Rightarrow$ $P(X > 381) = 0.025$ (symmetry), so 381 is the maximum acceptable Acceptable range is 368mL to 381mL	3 marks 2 marks for finding either the upper limit or the lower limit, with working 1 mark for finding the z-score of 382, or otherwise demonstrating some understanding of the normal distribution
34a	$h = k\cos(bt) + c$ The min height is 10 and the max height is 110, so the amplitude is 50 and the centre is 60, so $c = 60$. One cycle is completed between $t = 0$ and $t = 30$, so the period is 30. Therefore $\frac{2\pi}{b} = 30 \Rightarrow 2\pi = 30b \Rightarrow b = \frac{\pi}{15}$. Since the initial height is a minimum, the value of k is negative. So $k = -50$. Note the graph of h vs. t is: 	2 marks all correct 1 mark for any correct value
34b	Find the times when the height is 85m	3 marks 2 marks for one correct interval (e.g. 12.40-12.50pm), or

	$85 = 60 - 50\cos\left(\frac{\pi t}{15}\right), \quad 0 \leq t \leq 60$ $25 = -50\cos\left(\frac{\pi t}{15}\right)$ $\cos\left(\frac{\pi t}{15}\right) = -\frac{1}{2}, \quad 0 \leq \frac{\pi t}{15} \leq 4\pi$ $\frac{\pi t}{15} = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ $t = 10, 20, 40, 50$ Looking at the graph, h will exceed 85 when t is between 10-20 mins and between 40-50 mins.  This corresponds to 12.40-12.50pm and 1.10-1.20pm	for finding all four relevant times 1 mark for finding the correct t-values
35	The PDF is shown below:  Area under curve = $10k$ $10k = 1 \Rightarrow k = 0.1$ The probability of a bus requiring a wait time of more than 7 minutes is: $P(X > 7) = 3 \times 0.1 = 0.3$ When catching two buses, the probability of at least one requiring a wait time of more than 7 minutes is: P(at least one wait time is more than 7) $= 1 - P$(neither wait time is more than 7) $= 1 - [P(X \leq 7)]^2$ $= 1 - P$(both wait times are less than or equal to 7) $= 1 - 0.7^2$ $= 0.51$	3 marks 1st mark for finding $k = 0.1$ 2nd mark for finding the probability of one bus wait time being more/less than 7 mins
36	The areas are equal, so when integrating from 0 to k, the area below the x-axis will cancel out the area above the x-axis:	3 marks for correct solution 1st mark for writing a correct

	$\int_0^k (x^2 - 1)\, dx = 0$ $\left[\frac{x^3}{3} - x\right]_0^k = 0$ $\frac{k^3}{3} - k = 0$ $k^3 - 3k = 0$ $k(k^2 - 3) = 0$ $k = 0, \pm\sqrt{3}$ but $k > 1$, $\therefore k = \sqrt{3}$ **Alternative solution:** $\left\lvert \int_0^1 (x^2 - 1)\, dx \right\rvert = \int_0^k (x^2 - 1)\, dx$ $\left\lvert \left[\frac{x^3}{3} - x\right]_0^1 \right\rvert = \left[\frac{x^3}{3} - x\right]_1^k$ $\left\lvert \frac{1}{3} - 1 \right\rvert = \frac{k^3}{3} - k - \left(\frac{1}{3} - 1\right)$ $\frac{2}{3} = \frac{k^3}{3} - k + \frac{2}{3}$ $\frac{k^3}{3} - k = 0$ etc $\Rightarrow k = \sqrt{3}$	equation involving integrals 2nd mark for integrating $x^2 - 1$ 3rd mark for finding the value of k. Note: you had to justify why k is not 0 or -sqrt3
37	The arithmetic sequence is 1, 1+d, 1+2d, … The geometric sequence is 1, r, r^2, … Second terms are equal, so $r = 1 + d$ 14th term of AS = 3 times 3rd term of GS, so $1 + 13d = r^2$ Solving simultaneously: $1 + 13d = 3(1 + d)^2$ $1 + 13d = 3 + 6d + 3d^2$ $3d^2 - 7d + 2 = 0$ $(3d - 1)(d - 2) = 0$ $d = \frac{1}{3}$ or 2 d being 1/3 would result in some terms of the arithmetic sequence not being integers (e.g. the second term would be 4/3). Therefore $d = 2$. Hence $r = 1 + 2 = 3$	4 marks for correct solution 1st mark for correctly using one of the series formulas, e.g. obtaining $T_2 = 1 + d$ 2nd mark for obtaining both equations connecting r and d 3rd mark for solving simultaneously to obtain the two values of r or d

Booklet 4 general notes:

- Be aware of overall time management. You don't have to do the paper in order.

- If a question says "Find the expected value of [Variable]", make a table of values where the top row is the possible values of [Variable] and the second row is the probability of each outcome.

  Question says "Rolling an even number = don't move. Rolling an odd number = move forward that number of spaces

x	0	1	3	5
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- Statistics 3- or 4-mark "Explain" questions are serious work. Make sure you address all of the given information (r=0 and y=390) and link these to the overall context (smoothie sales scatter plot). Avoid vague or general statements.

- For normal distribution empirical rule questions, show all working / write reasons for everything. e.g. "1% of cans are more than 382mL, therefore 1% of cans are less than 368mL (symmetry)".

- $y = \sin(bx)$ $\Rightarrow$ Period = $\frac{2\pi}{b}$. This formula is the key to solving many trig problems.

- Trig graphs:.k = -50 tripped up a lot of students (negative cosine shape with amplitude 50)

- Sometimes drawing a graph/ diagram can really help your working

  e.g. carriage pod question, bus stop question

- P(at least one...) = 1 – P(none...)

- If an area is under the x-axis, the integral is negative. Use absolute values. e.g.
  $Area = \left| \int_0^1 (x^2 - 1)\, dx \right|$.
  They love testing this concept in the HSC.

- When solving equations, dividing by x can cause problems. Use factorisation instead. e.g. solve $x^2 = 5x$

- Know your series formulas and when to use them. General term and sum of an AP and a GP.